Convex hulls in 2D

The problem: Given a set P of n points in the plane, find their convex hull.

Algorithm: Graham scan

Idea: start from a point p interior to the hull. Order all points by their ccw angle wrt p and traverse them. Maintain the CH of all points traversed so far and add the next point to it.

Algorithm GrahamScan (input: points P)

- Find interior point p_0 (instead of an interior point, can pick the lowest point)
- Sort all other points ccw around p_0; denote them $p_1, p_2, ...p_{n-1}$ in this order.
- Initialize stack $S = (p_2, p_1)$
- for i = 3 to n-1 do
  - if p_i is left of (second(S), first(S)): push p_i on S
  - else:
    - repeat: pop S while p_i is right of (second(S), first(S))
    - push p_i on S

Questions:

1. Run Graham-scan on a small set of points and check how it works. Assume no "degenerate" cases.
2. Argue that once the points are sorted, the algorithm takes linear time.
3. What are the degenerate cases for Graham-scan, and how do you extend the algorithm to handle these cases?